Momentum Trading and Performance with Wrong Return Expectations

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The investment performance of many long-term institutional investors is evaluated in the interim, for example, using annual returns. The *optimum-growth* policy, which maximizes expected net asset value in the long run, is a natural choice for long-term institutional investors, such as endowments, pension funds, and many mutual funds. In the case of constant investment opportunities, the policy specifies that constant fractions of wealth are invested in different asset classes. This is manifested in the strategic asset allocations of institutional investors. Deviations from the strategic allocation, called tactical asset allocation, reflect private assumptions about the predictability of returns for particular asset classes.

This article examines the expected return and dynamic trading for the optimum-growth portfolio policy under wrong assumptions about changing expected returns. We show that a misspecified policy under fixed wrong beliefs can outperform the correct one in terms of annual expected return. However, the characteristics of the private assumptions about how expected returns are changing can be identified from observed rebalancing of a portfolio. In particular, assuming that expected returns change due to current mispricing makes returns predictable in the short run and can lead to optimal momentum trading, which calls for buying more shares after the price has increased. In contrast, contrarian trading that calls for selling shares after price increases is always optimal when returns are unpredictable, because prices are assumed to follow a random walk around a long-term trend.

We consider subjective beliefs in the general sense of assumptions about the specification of asset price dynamics. It seems implausible, considering differences of opinion about parameter estimates, that investors can know expected returns remain constant through time, while they are unable to compute unbiased estimators for a constant mean parameter. In contrast, we assume that the dynamic specification for the mean can not be readily ascertained (see, for example, Merton [1980], Summers [1986], and Poterba and Summers [1988]). Moreover, in our analysis, the private dynamic specifications used by investors are not updated over a certain period of time because learning is slow and costly, and a new specification is adopted only after enough data have been collected to conclusively reject the current specification. Finally, our investors do not incorporate uncertainty about the assumed specification and the estimated parameters into their portfolio optimization. Our goal is to characterize the basic portfolio optimization that is taught in business schools and is subsequently practiced by many investment professionals.

The simple intuition of this article is that investors who are wrong about returns being unpredictable do not time the changing investment opportunities; this can result in high risk taking relative to the correct optimal policy. Spurious timing is optimal, however, when investors are wrong in assuming predictable returns, and prices actually follow a random walk. In both cases, wrong assumptions about asset price dynamics cause suboptimally high risk taking over certain price paths, which can lead to expected annual returns that are higher than those of the correctly specified policy.

But the optimal dynamic trading strategy with time-varying expected returns is qualitatively different from the strategy when expected returns are constant. Assuming that expected returns are changing, and returns are therefore predictable, implies that the characteristics of the optimal trading strategy depend on the particular price-path realization. Certain paths imply that momentum trading is the optimal trading strategy. As a result, continuous monitoring of a portfolio's rebalancing can reveal a manager's private assumptions about return predictability.

We consider two types of asset price dynamics: predictable returns when prices are reverting toward a long-term trend and expected returns are changing, and the unpredictable random walk when prices are log-normally distributed, as in the Black–Scholes model. In this way, we can compare the two alternatives in terms of trading and expected returns of using the wrong assumptions: false predictability versus ignoring predictability.

Comparing the expected annual returns from dynamic trading implied by the alternative types of misspecification (assuming predictability rather than ignoring it) reveals that misspecified return predictability leads to expected annual return that is decreasing for suffciently large mispricing, both for misspecified overpricing and for misspecified underpricing. In contrast, using a misspecified constant drift instead of the correctly specified trend-reverting one implies an expected annual return that is *increasing* as the underpricing is increasing.

The higher expected returns under wrong assumptions, however, are not simply driven by higher risk taking, but depend on the realized price-path, and hence the expected returns are not always increasing with risk. We show that suffciently extreme beliefs always lead to expected underperformance of the misspecified policy.

Long-term investing with predictability has been studied by Barberis [2000] and others. Our innovation is to analyze explicitly the impact of misspecified predictability on the expected annual performance and on the observable dynamic trading of long-term institutional investors. Our results are also related to some recent entries in the revealed preferences literature. For example, Dybvig and Rogers [1997] studied the feasibility of recovering preferences from observed optimal consumption policies when prices follow a random walk and investment opportunities remain constant. We focus on the trading and investment performance that depends on the realized price path under a fixed misspecification of dynamically changing investment opportunities.

Our results can be used to inform the principals, for example, investment committees, in the annual performance of institutions, such as endowments, pension plans, mutual funds, and hedge funds, that have long-term investment objectives. Our analysis highlights the importance of discussing a portfolio's rebalancing over an investment period in relation to asset class performance. Such analysis can be helpful for the identification of extreme false assumptions.

OPTIMUM GROWTH AND RETURN PREDICTABILITY

Consider investors who are price takers over an investment horizon, $T = 1$, and assume the risk-free rate, r, is constant. Consider one risky asset, with price P at time t, which can be interpreted as the asset class mandate of an institutional investor. The asset class returns can be predictable in the form of reversion toward a long-term trend. Denote $p \equiv \ln P$. The instantaneous return at time t has a conditional expected return, μ, that reflects expected reversion to a long-term trend, γ,

$$\frac{dP}{P} = \mu dt + \sigma dz \tag{1}$$

$$\mu \equiv \gamma - \alpha(p - \gamma t)$$

The speed of expected reversion to the trend is determined by the parameter $\alpha > 0$. It determines whether the implied returns are predictable: if $\alpha > 0$, then returns are predictable because a mispricing correction is expected. Investors who believe that there is current mispricing adjust their portfolios to take advantage of the perceived mispricing, as in the case of tactical asset allocation. Investors who specify large values for α believe that the mispricing will be corrected quickly, so the short-term expected return has large absolute value. When $\alpha = 0$, this is merely the traditional Black–Scholes log-normal prices with drift γ, and prices follow a random walk with a drift. It is convenient to denote by X the current mispricing from the long-term trend, γ,

$$X_t \equiv p(t) - \gamma t \tag{2}$$

The optimum-growth portfolio policy maximizes expected long-term wealth (see Merton [1971]). The optimum-growth portfolio policy merely takes into account the current investment opportunity set with no additional hedging demands,

$$\omega_t = \frac{\mu(X,t) - r}{\sigma^2} \tag{3}$$

When prices are assumed to follow a random walk, as for the log-normal specification with $\alpha = 0$, the policy dictates investing a constant fraction of wealth in the risky asset,

$$\bar{\omega} \equiv \frac{\gamma - r}{\sigma^2} \tag{4}$$

This can be interpreted as the strategic asset allocation of long-term investors. In contrast, for the predictable returns that have $\alpha > 0$, the optimum-growth policy is price-path dependent, because it depends on the current mispricing, $X(t)$,

$$\omega_t = \bar{\omega} - bX_t \tag{5}$$

$$b = \frac{\alpha}{\sigma^2}$$

This corresponds to tactical asset allocation that responds to current developments in the markets. We consider next the annual performance of misspecified and correctly specified portfolio policies.

EXPECTED ANNUAL RETURNS

The standard dynamic equations can be expressed in terms of the current mispricing, X, that was previously defined. Now the continuously compounded average rate of return at time $T = 1$, interpreted as the annual return, is

$$R \equiv \ln\left(\frac{W_1}{W_0}\right) \tag{6}$$

For simplicity, we set $W_0 = 1$. We now evaluate the expected annual return for the misspecified versus correctly specified optimum-growth policies under the two alternative scenarios for the predictable and unpredictable asset class returns.

Random Walk: Asset Class with Log-normal Price Process

The random walk is obtained for $\alpha = 0$ in the trend-reverting specification, $dX = \sigma dz$, and the mispricing, X_t, is a normally distributed deviation from the trend,

$$X_t \sim N(X_0, \sigma^2 t) \tag{7}$$

Correctly Specified Constant Allocation Policy

The correctly specified policy invests a constant fraction of wealth, $\omega_t = \bar{\omega}$, in the asset, and the terminal wealth is path independent,

$$W_1 = e^{(1-\bar{\omega})\left(r + \bar{\omega}\frac{\sigma^2}{2}\right)} \left(\frac{P_1}{P_0}\right)^{\bar{\omega}} \tag{8}$$

The annual return is normally distributed with an expected value of

$$E[R] = r + \frac{1}{2}\left(\frac{\gamma - r}{\sigma}\right)^2 \tag{9}$$

Misspecified Policy Assuming Trend Reversion

The annual return of the misspecified policy is path dependent due to the path-dependent optimal portfolio, $\omega_t = \bar{\omega} - bX_t$. Intuitively, the misspecified policy assumes trend reversion, and invests more in the risky asset when it is more underpriced and less when it is overpriced. The expected annual return, is[1]

$$E[R] = J(\alpha)X_0^2 + K(\alpha)X_0 + L(\alpha) \tag{10}$$

$$J(\alpha) < 0,\ K(\alpha) < 0$$

Note that the negative signs of the coeffcients $K(\alpha)$, $J(\alpha)$ do *not* depend on the investor's beliefs about the trend-reversion speed, α. The expected annual return is a quadratic function of the current misspecified subjective mispricing, X_0. For moderate mispricing, $X_0 \approx 0$, the linear term $K < 0$ is driving the expected return. As the subjective overpricing, $X_0 > 0$, increases, the expected return decreases (since $K(\alpha) < 0$), while more subjective underpricing, $X_0 < 0$, implies that the expected return first increases, and then ultimately decreases for P_0 close to zero. Moreover, since $K(\alpha) \uparrow 0$, the expected return is not maximized when the investor believes that there is no current mispricing, $X_0 = 0$.

Expected Interim Performance When Price Follows a Random Walk

For a log-normal asset class, the misspecified return predictability that we consider has two aspects: the mispricing, X_0, and the rate, α, at which the log-price reverts to the trend, γ. To separate the two effects, we fix the reversion speed, α, over the time period $T = 1$, and examine the comparative static of the expected annual return with respect to the subjective mispricing, X_0. We can prove the following result.

Proposition 1. There is a range of values for the subjective underpricing, X_0, for which the expected annual return is *increasing* with increasingly negative subjective underpricing. The expected annual return is decreasing for suffciently large absolute values of the mispricing, $|X_0| > X_0^*$.

The proposition shows that higher realized annual returns may reflect investing based on wrong assumptions about predictability. This result is counterintuitive if using wrong assumptions about more extreme negative mispricing "should not" lead to higher performance on average. The intuition of this result is that the expected annual return is increasing with more negative mispricing because of higher risk taking. We now compare the expected annual return of the misspecified policy to that of the correctly specified one. We can prove the following result.

Proposition 2. There is a range of non-extreme subjective beliefs for which the expected annual return of the misspecified trend-reverting policy is higher than the expected return of the correct log-normal policy. This is true even if there is no subjective current mispricing.

This result also seems surprising if false assumptions "should not" produce higher expected returns than the correct ones. The intuition of this result is that more incorrect assumptions lead to more risk taking and are rewarded with higher expected return.

Predictable Returns: Trend-Reverting Asset Class

For trend reversion, we have $\alpha > 0$ in the specification for the mispricing, $dX = -\alpha X dt + \sigma dz$, and the mispricing, X_t, at time t is a normally distributed deviation from the trend,

$$X_t \sim N\left(X_0 e^{-\alpha t}, \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha t}) \right) \tag{11}$$

We consider the correct trend-reverting policy first.

Correctly Specified Trend-Reverting Policy

Substituting the path-dependent optimal portfolio, $\omega_t = \bar{\omega} - bX_t$, $0 \le t \le 1$, in the return expression in Equation (10), we can show that the expected return is

$$E[R] = F(\alpha)X_0^2 + G(\alpha)X_0 + I(\alpha) \tag{12}$$

$$F(\alpha) > 0$$

The positive sign of the coefficient $F(\alpha)$ does not depend on the investor's assumptions about the speed, α, or the current mispricing, X_0. The expected annual return $E[R]$ is a quadratic function of the current mispricing, X_0. This is a consequence of both the asset dynamics and the optimal portfolio policy being linear functions of the mispricing. Since the sign of the quadratic coefficient $F(\alpha)$ is positive, higher absolute mispricing implies higher expected returns. Intuitively, the correctly specified policy exploits any mispricing, and higher mispricing implies higher current drift of the asset price and higher expected annual return.

Misspecified Constant Allocation Policy

For a predictable asset class, the misspecified log-normal policy ignores the predictability, and the fraction invested in the risky asset, $\omega = \bar{\omega}$, is constant. However, the annual return is still price-path dependent due to the trend reversion of the prices. Substituting the optimal policy in the return expression in Equation (12), we can show that the return is

$$R = r + \frac{1}{2}\sigma^2\bar{\omega}^2 + \bar{\omega}[X_1 - X_0] \tag{13}$$

The realized annual return is higher when the overpricing has increased for the realized path. Intuitively, the paths have negative current drift if there is positive overpricing at time 0, while a price-path realization with an increase in overpricing corresponds to an average positive drift. In this case, the misspecified policy that is using a misspecified positive subjective drift realizes higher annual returns for the unlikely price-path realization.

Since the mispricing at the end of the performance evaluation period, X_1, is normally distributed, the annual return is normally distributed with mean

$$E[R] = r + \frac{1}{2}\sigma^2\bar{\omega}^2 - \bar{\omega}(1 - e^{-\alpha})X_0 \tag{14}$$

Note that the expected annual return is a linear function of the mispricing, X_0. This leads to an asymmetric dependence of the misspecified expected return on the subjective mispricing. Because the linear coefficient is negative, we have the following result.

Proposition 3. The expected annual return of the misspecified policy increases with more negative underpricing, $X_0 < 0$, and decreases with increasing overpricing, $X_0 > 0$.

Intuitively, increasing overpricing implies *lower* (and eventually negative) drift, but the misspecified policy ignores it and does not decrease the holdings of the risky asset. As a result, the constant risky fraction of the portfolio earns a lower expected return. Conversely, more negative underpricing implies higher drift, and the constant fraction of wealth that is invested in the risky asset earns a higher expected return.

Expected Interim Performance When Price Is Trend Reverting

The expected annual return of the correct policy increases in absolute mispricing, while the misspecified expected return increases only with more (negative) underpricing. Correctly timing the predictable asset class—when the price is too low and when it is too high—implies a higher return, but incorrect beliefs that the asset class is unpredictable and has positive expected return leads to lower expected return when the asset is overpriced. We now compare the performance of the two policies.

Proposition 4. There is a non-extreme range of parameter values for which the expected annual return of the misspecified log-normal policy is higher than the expected return of the correct path-dependent policy.

This apparently counterintuitive result shows that the misspecified constant allocation policy can outperform the correct one that assumes predictable returns. Note, however, that as current mispricing increases, the quadratic expected annual return of the correct policy eventually dominates the misspecified expected return even for negative underpricing.

MOMENTUM AND CONTRARIAN TRADING

We characterize the optimum-growth portfolio policy in terms of momentum versus contrarian trading. Let $n = wW/P$ be the number of shares priced at P per share. We introduce the following definition of momentum trading.

Momentum Trading

$$\Delta P > (<)\ 0 \quad \text{implies } \Delta n > (<) 0$$

Contrarian Trading

$$\Delta P > (<)\ 0 \quad \text{implies } \Delta n < (>) 0$$

According to this definition, momentum traders buy more shares as the price increases, and contrarians buy more shares as the price decreases.

Consider the two different portfolio policies discussed in the first section. An investor who assumes that prices follow a random walk optimally invests a constant fraction of wealth in the risky asset. Yet, the assumption of trend reversion implies a path-dependent optimal portfolio policy.

A constant portfolio weight, $\omega = \bar{\omega}$, is, for example, the optimal portfolio for random walk.

Proposition 5. An investor who invests a constant, unlevered fraction of wealth in the risky asset is always a contrarian trader, regardless of the particular specification for the mean of the returns. Conversely, a levered investor is always a momentum trader.

The path-dependent portfolio weight w that is linear in the current mispricing, X_t, is, for example, the optimal portfolio for predictable returns,

$$\omega(t) = A(t) - B(t)X_t, \;\; B(t) > 0 \tag{15}$$

for example, $A(t) = a(t)$, $B(t) = b(t)$ as considered in Equation (15). The conditions for momentum trading can be expressed using the subjective mispricing, X_t. We can prove the following result.

Proposition 6. An investor who invests a path-dependent fraction of wealth in the risky asset, $\omega = A - BX$, executes a momentum trade when

$$X_t < M(A,B) \;\text{ or }\; \frac{A}{B} < X_t < N(A,B) \tag{16}$$

$$M(A,B) \equiv \frac{A}{B} - \frac{\sqrt{1+4B}+1}{2B}, \; N(A,B) \equiv \frac{A}{B} + \frac{\sqrt{1+4B}-1}{2B}$$

Note that for a fixed mean-reversion parameter, α, the path-dependent policy leads to momentum trading for a range of subjective mispricing parameters, X_t. It can be shown that the result is not an "exotic" scenario requiring extreme parameter values. In contrast, the fixed-fraction policy is always contrarian.

Path-dependent portfolio policies, such as the one optimal for predictable returns, imply momentum trading, because the short-term changes in the investment opportunity set imply optimal changes in the risky holdings. For example, in the case of high underpricing and high expected short-term return, a realized mispricing correction, which is slower than the expected one, means that the price increases, but the expected short-term return increases even further, implying higher holdings of the risky asset. This is manifested as momentum trading. In other words, if the expected price correction is not fully realized, the investment can look even more attractive and the holdings are optimally increased.

CONCLUSION

This article has characterized the expected annual performance and dynamic trading of the optimum-growth policy based on subjective and possibly incorrect assumptions about return predictability. We considered two types of asset price dynamics: trend-reverting prices that imply return predictability, and an unpredictable log-normal random walk. We characterized the expected annual investment performance of the policies in terms of expected holding-period return. We derived conditions for misspecified policies to have expected annual returns that are higher than the expected return of the correctly specified policy.

The portfolio policies were compared in terms of momentum versus contrarian trading. We showed that a nonlevered investor who assumes prices follow a random walk always trades as a contrarian, buying more shares as the price decreases and selling shares when the price increases. In contrast, assuming price predictability can lead to momentum trading, which means buying as the price increases. These results imply that observed continuous trading can shed light on the assumptions about return predictability.

ENDNOTE

[1]The proofs of the following results are available upon request.

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